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AMBIGUITY FUNCTION ANALYSIS FOR THE HYBRID MIMO PHASED-ARRAY RADAR (POSTPRINT)

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**Radar Signal Processing Technology Branch
RF Sensor Technology Division**

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AMBIGUITY FUNCTION ANALYSIS FOR THE HYBRID MIMO PHASED-ARRAY RADAR

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ABSTRACT

The Hybrid MIMO Phased Array Radar, or HMPAR, is a notional concept for a multisensor radar architecture that combines elements of traditional phased-array radar with the emerging technology of Multiple-Input Multiple Output (MIMO) radar. A HMPAR comprises a large number, MP , of T/R elements, organized into M subarrays of P elements each. Within each subarray, passive element-level phase shifting is used to steer transmit and receive beams in some desired fashion. Each of the M subarrays are in turn driven by independently amplified phase-coded signals. This paper derives a version of the radar ambiguity function that is appropriate for this radar architecture. The ambiguity function is a function of time delay, Doppler frequency shift, and two or more spatial variables. An illustrative example for a particular MIMO signal set is given.

1. INTRODUCTION

Multiple-Input Multiple Output, or MIMO, radar systems are next-generation radar systems with multiple transmit and receive apertures, equipped with the capability of transmitting arbitrary and differing signals at each transmit aperture. The enabling technologies for such systems include all-digital radars with high-speed (GHz) A/D and D/A converters, arbitrary waveform generators, and the ever-increasing speed and memory of the computers and embedded processors. MIMO radar systems are often contrasted with phased-array radars, that transmit the same signal at each aperture, shifted by an arbitrary phase using analog electronics. The added flexibility of individual signal selection at each aperture brings with it the promise of enormous performance improvements, and the challenge of finding solutions to extremely high-dimensional

optimization problems associated with choosing the right signals. See [1] and the many references therein.

In this report we consider a variation of the MIMO radar concept for colocated sensor assets that we term the Hybrid MIMO Phased Array Radar, or HMPAR, first proposed by Browning *et al.* [2]. The HMPAR concept brings together elements of both MIMO and phased-array radar. There are a large number MP of T/R elements, organized into M subarrays of P elements each. This is illustrated in Figure 1.1, which depicts the notational concept for the HMPAR with elements arranged in a rectangular array. Other configurations (*e.g.* hexagonal) are possible but are not considered here.

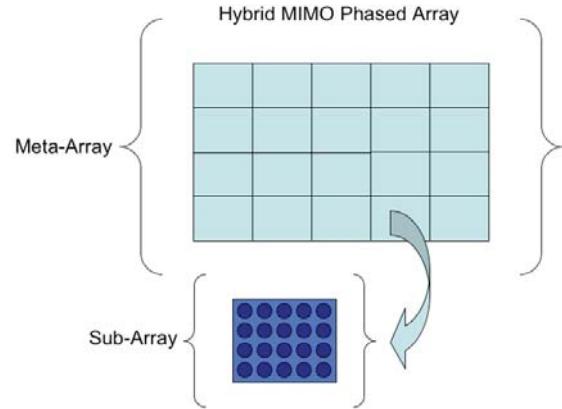


Figure 1.1 HMPAR notional concept.

Within each subarray, passive element-level phase shifting is used to steer transmit and receive beams in some desired fashion. Each of the M subarrays are in turn driven by independently amplified phase-coded signals which could be quasi-orthogonal, phase-coherent, or partially correlated. In Browning's original concept, such a radar system could be an electronically steered planar array deployed in an airborne platform, *e.g.* in the radome of a fighter aircraft, for concurrent search, detect, and track missions. The objective of the ongoing research described

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in part here is to identify transmit signaling strategies and adaptive receive signal processing algorithms consistent with these requirements.

In previous papers [2,3] we described in more detail the general HMPAR concept and transmit signaling strategies that could be employed to achieve arbitrary spatial transmit beampatterns. The bulk of this paper is devoted to the derivation of a version of the radar ambiguity function that is appropriate for this particular radar architecture. It will be shown that the ambiguity function is a function of time delay, Doppler frequency shift, and two or more spatial variables. In Section 3, an illustrative example based on the transmit signaling strategies described in [3] is given. Our example demonstrates that with this particular radar architecture, as is true with MIMO radar in general, one can achieve phased-array-like resolution on receive with arbitrary spatial beampatterns on transmit.

2. HMPAR AMBIGUITY FUNCTION

In this section we develop the expressions that will be used in evaluating the signal sets used to drive the HMPAR transmitters. The full MIMO ambiguity function is derived in San Antonio, Fuhrmann, and Robey [4]. Here, because the assets are colocated, the ambiguity function can be expressed in less than 12 arguments as is required in the most general case. The results here are closely related to Case 3 of [4], except that we take into account the transmit and receive beampatterns of the phased subarrays as well. In order to be self-contained, we derive the desired ambiguity function from first principles.

The underlying meaning or interpretation of the radar ambiguity function varies somewhat in the literature. We define it as follows: for a given transmitted signal $s(t)$, or signal set expressed in vector form as $\mathbf{s}(t)$, there is a received signal set denoted $\mathbf{x}(t)$ which is a function of the point target parameters, denoted θ . The received signal set $\mathbf{x}(t)$ is normalized to unit norm, in an appropriate norm, so that the ambiguity function is not a function of path losses or target cross-section. Then the ambiguity function for transmitted signal set $\mathbf{s}(t)$ is defined as the inner product of the received signals under two different sets of target parameters:

$$\chi_s(\theta_1, \theta_2) = \int_{-\infty}^{\infty} \text{tr} \mathbf{x}(t; \theta_1) \mathbf{x}^H(t; \theta_2) dt . \quad (2.1)$$

Clearly the ambiguity function is maximized at 1 for $\theta_1 = \theta_2$. In the original formulation of Woodward [5], the target parameters are expressed in radar coordinates of delay and Doppler. In the most general case, the target parameters would be position and velocity in three dimensions. Here we will see an intermediate case between

these two, with the target parameters being range, Doppler, and spatial angle in one or two dimensions.

We begin the derivation by looking at a single signal $s_T(t)$ transmitted from a monostatic radar system, and a target located at a range corresponding to a round-trip delay of τ . The signal is given in its analytic (complex) form. The signal propagates through space and impinges on the target after delay $\frac{\tau}{2}$. The signal as seen at the target is denoted

$$s_1(t) = s_T(t - \tau/2) . \quad (2.2)$$

The moving target imparts a Doppler shift to the signal; we can model this mathematically by imagining that the target multiplies the signal by a complex exponential, then re-radiates the signal back toward the radar. The re-radiated signal is

$$s_2(t) = s_T(t - \tau/2) e^{j2\pi f_D(t - \tau/2)} . \quad (2.3)$$

Here the phase of the complex exponential is chosen to make the phase equal to 0 at the point where the leading edge of the transmitted signal reaches the target. The re-radiated signal propagates back to the radar and experiences a time delay of $\frac{\tau}{2}$, so that the received signal is

$$s_3(t) = s_2(t - \tau/2) = s_T(t - \tau) e^{j2\pi f_D(t - \tau)} . \quad (2.4)$$

Now we write the transmitted signal as a product of a baseband envelope and a carrier term:

$$s_T(t) = s(t) e^{j2\pi f_c t} . \quad (2.5)$$

Then

$$s_3(t) = s(t - \tau) e^{j f_c(t - \tau)} e^{j2\pi f_D(t - \tau)} \quad (2.6a)$$

$$= s(t - \tau) e^{j2\pi f_D t} e^{-j2\pi(f_c + f_D)\tau} e^{j2\pi f_c t} . \quad (2.6b)$$

The envelope of the received signal is

$$\tilde{s}_3(t) = s(t - \tau) e^{j2\pi f_D t} \left(e^{-j2\pi(f_c + f_D)\tau} \right) . \quad (2.7)$$

The term in parentheses is a bulk phase term that can be ignored, since it could be included in the random phase of the carrier. The phase of the ambiguity function is generally ignored as well. Working solely with the complex baseband envelope of the received signal, denoted $x(t)$, we have

$$x(t) = s(t - \tau) e^{j2\pi f_D t} . \quad (2.8)$$

Now for the same transmitted signal suppose we have two received signals experiencing different time delays τ_1 and τ_2 , and different Doppler frequency shifts f_1 and f_2 :

$$x_1(t) = s(t - \tau_1)e^{j2\pi f_1 t} \quad (2.9a)$$

$$x_2(t) = s(t - \tau_2)e^{j2\pi f_2 t} \quad (2.9b)$$

The ambiguity function is defined as

$$\chi(\tau_1, \tau_2, f_1, f_2) = \int_{-\infty}^{\infty} x_1(t)x_2^*(t)dt \quad (2.10a)$$

$$= \int_{-\infty}^{\infty} s(t - \tau_1)s^*(t - \tau_2)e^{j2\pi(f_1 - f_2)t}dt \quad (2.10b)$$

Under the change of variables $\tau = \tau_1 - \tau_2$ and $f = f_1 - f_2$, and again ignoring any bulk phase terms that result, we have

$$\chi(\tau, f) = \int_{-\infty}^{\infty} s(t)s^*(t + \tau)e^{j2\pi f t}dt \quad (2.11)$$

We will use (2.11) as our baseline definition of the ambiguity function for a single radar signal in a monostatic setting. Note that generally only $|\chi(\tau, f)|$ or $|\chi(\tau, f)|^2$ are of interest.

Now suppose we have two different signals $s_1(t)$ and $s_2(t)$. The correlation of $s_1(t)$ (experiencing delay τ_1 and frequency shift f_1) and $s_2(t)$ (experiencing τ_2 and f_2) is

$$\chi_{12}(\tau, f) = \int_{-\infty}^{\infty} s_1(t)s_2^*(t + \tau)e^{j2\pi f t}dt \quad (2.12)$$

This is called the cross-ambiguity function, or CAF.

When there are M different signals $s_1(t) \dots s_M(t)$ stacked into a vector $\mathbf{s}(t)$, we can organize all the CAFs into an $M \times M$ matrix which termed the matrix cross-ambiguity function, or MCAF:

$$\chi_M(\tau, f) = \int_{-\infty}^{\infty} \mathbf{s}(t)\mathbf{s}^H(t - \tau)e^{j2\pi f t}dt \quad (2.13)$$

Note that

$$\chi_M(\tau, 0) = \mathbf{R}_x(\tau) \quad (2.14)$$

is the transmitted signal cross-correlation matrix. $\mathbf{R}_x(0)$ is the zero-delay signal cross-correlation matrix, which controls the transmitted beampattern in a MIMO radar system.

We now turn our attention to the development of an ambiguity function for the HMPAR architecture. Suppose that there are a total of MP transmitting elements, organized into M subarrays of P elements each. Each subarray i , $i = 1 \dots M$, has a local phase center x_i and a spatial response pattern $b_i(\theta)$, including any element patterns, each defined relative to the respective phase centers. $b_i(\theta)$ is determined by passive phase shifts applied to each

of the elements. In addition, define a global phase center for the entire array, denoted x_0 .

Define the *meta-array* as the M -element array of hypothetical omnidirectional sensors located at the local phase centers of the subarrays. The meta-array has a response vector, defined relative to the global phase center x_0 , that can be denoted

$$\mathbf{a}(\theta) = \begin{bmatrix} a_1(\theta) \\ \vdots \\ a_M(\theta) \end{bmatrix} \quad (2.15)$$

The response for each subarray, now defined relative to the global phase center, is

$$c_i(\theta) = a_i(\theta)b_i(\theta) \quad (2.16)$$

and again these can be collected into a signal vector:

$$\mathbf{c}(\theta) = \begin{bmatrix} c_1(\theta) \\ \vdots \\ c_M(\theta) \end{bmatrix} \quad (2.17)$$

Suppose that each subarray is driven by a different signal $s_i(t)$, and that the collection of all M signals propagates toward a point target. Because of the colocated asset model, we may assume that all signals experience the same bulk time delay and frequency shift (differential time delays are already accounted for in the spatial response patterns.) Ignoring path losses, the signal at the target location due to transmitter i is

$$y_i(t) = b_i(\theta)a_i(\theta)s_i(t - \tau/2) \quad (2.18)$$

and the superposition of all such signals is

$$y(t) = \sum_{i=1}^M b_i(\theta)a_i(\theta)s_i(t - \tau/2) \quad (2.19)$$

where the time delay τ is defined relative to the global phase center. If the target is moving, a single Doppler shift is imparted to the composite incident signal:

$$\tilde{y}(t) = y(t)e^{j2\pi f(t-\tau/2)} \quad (2.20)$$

$$= \sum_{i=1}^M c_i(\theta)s_i(t - \tau/2)e^{j2\pi f(t-\tau/2)}.$$

As in the earlier development, model the reflection as a re-radiation of the signal $\tilde{y}(t)$ back toward the radar system. In this case, each subarray j responds to the composite reflected signal. Assuming reciprocity on transmit and receive for the passive phase shifting (each subarray produces a scalar output), these received signals can be written

$$r_j(t) = \tilde{y}(t - \tau/2)c_j(\theta) \quad (2.21a)$$

$$= \sum_{i=1}^M c_i(\theta)c_j(\theta)s_i(t - \tau)e^{j2\pi f(t-\tau)} \quad (2.21b)$$

Stacking all the received signals into a signal vector, we have

$$\mathbf{r}(t) = \begin{bmatrix} r_1(t) \\ \vdots \\ r_M(t) \end{bmatrix} \quad (2.22a)$$

$$= \mathbf{c}(\theta)\mathbf{c}^T(\theta)s(t - \tau)e^{j2\pi f(t-\tau)} \quad (2.22b)$$

The space-time HMPAR ambiguity function χ_H will be defined as the inner product of $\mathbf{r}(t)$ for one set of target parameters with some $\mathbf{r}(t)$ for another:

$$\chi_H(t_1, t_2, f_1, f_2, \theta_1, \theta_2) \quad (2.23)$$

$$\begin{aligned} &= \text{tr} \int_{-\infty}^{\infty} \mathbf{r}(t; \tau_1, f_1, \theta_1) \mathbf{r}^H(t; \tau_2, f_2, \theta_2) dt \\ &= \text{tr} \int_{-\infty}^{\infty} \mathbf{c}(\theta_1) \mathbf{c}^T(\theta_1) \mathbf{s}(t - \tau_1) e^{j2\pi f_1 t} e^{-j2\pi f_2 t} \mathbf{s}^H(\theta_2) \mathbf{c}^H(\theta_2) dt \\ &= \int_{-\infty}^{\infty} \mathbf{c}^T(\theta_1) \mathbf{s}(t - \tau_1) \mathbf{s}^H(t - \tau_2) e^{j2\pi(f_1 - f_2)t} \mathbf{c}^*(\theta_2) \mathbf{c}^H(\theta_2) \mathbf{c}(\theta_1) dt \\ &= \mathbf{c}^H(\theta_2) \mathbf{c}(\theta_1) \cdot \mathbf{c}^T \int_{-\infty}^{\infty} \mathbf{s}(t - \tau_1) \mathbf{s}^H(t - \tau_2) e^{j2\pi(f_1 - f_2)t} dt \mathbf{c}^*(\theta_2) \\ &= \mathbf{c}^H(\theta_2) \mathbf{c}(\theta_1) \cdot \mathbf{c}^T(\theta_1) \chi(\tau, f) \mathbf{c}^*(\theta_2) \end{aligned}$$

In this final expression, we see that χ_H factors naturally into two terms, which we call the *receive factor* $\mathbf{c}^H(\theta_2)\mathbf{c}(\theta_1)$ and the *transmit factor* $\mathbf{c}^T(\theta_1)\chi(\tau, f)\mathbf{c}^*(\theta_2)$. The transmit factor is the inner product of the total signal received at one target with that of another target (including any Doppler shifts). The receive factor is the inner product of the response vectors for two different angles, i.e. the beampattern in θ_2 for a signal steered at θ_1 .

Eqn. (2.33) is essentially the same as (33) in [4], except that in [4] there is an additional dependence of the array response on the shifted frequency due to the target motion, since array response does depend on the center frequency. This effect is not accounted for in our model, and is assumed negligible.

Note that χ_H depends on one delay and one Doppler variable, but two separate spatial variables θ_1 and θ_2 . This is a result of the fact that the ambiguity is a function only

of the *difference* between two time delays and two frequency shifts, whereas for sensor arrays one does not see the same sort of stationarity in the response for pairs of spatial angles.

Our result is also similar to, and consistent with, the expressions in Chen and Vaidyanathan [6]. They assume omnidirectional subarrays (i.e. there are no subarray patterns) and they also allow for the possibility of differing transmit and receive arrays.

$\chi_H(0, 0, \theta, \theta)$ is the *transmit beampattern* that may be controlled through the selection of the signal cross-correlation matrix.

$\chi_H(0, 0, \theta_1, \theta_2)$ is included in the cost function that Stoica and Li [7] attempt to minimize through the signal design. We remark that considering the angular correlations only, at zero delay and zero Doppler, does not capture all the effects one may one to consider in signal design and therefore we prefer to look at the full ambiguity function.

To summarize, here are the key equations derived in this section.

Subarray and Meta-Array Response:

$$\mathbf{c}(\theta) = \mathbf{a}(\theta) \circ \mathbf{b}(\theta) \quad (2.34)$$

where $\mathbf{b}_i(\theta)$ is the i th subarray pattern defined relative to the i th local phase center, and $a_i(\theta)$ is the free-space phase of the i th phase center relative to the global phase center.

Signal Matrix Cross-Ambiguity Function:

$$\chi_M(\tau, f) = \int_{-\infty}^{\infty} \mathbf{s}(t) \mathbf{s}^H(t + \tau) e^{j2\pi f t} dt \quad (2.35)$$

where $\tau = \tau_1 - \tau_2$ and $f = f_1 - f_2$.

HMPAR Ambiguity Function:

$$\chi_H(\tau, f, \theta_1, \theta_2) = \mathbf{c}^H(\theta_2) \mathbf{c}(\theta_1) \cdot \mathbf{c}^T \chi_M(\tau, f) \mathbf{c}^*(\theta_2) \quad (2.36)$$

3. AMBIGUITY FUNCTION DISPLAYS

In this section we present results on visualization of the MIMO radar ambiguity for one HMPAR scenario.

The signals used for illustration here are described in [3]. In summary, the $M \times N$ signal matrix \mathbf{S} is constructed by having the rows be complex exponential phase sequences, with a slight frequency offset Δf from row to row. The value of the Δf controls the signal cross-correlation and the transmit beamwidth. The columns of \mathbf{S} are multiplied pointwise by a pseudo-noise (PN) sequence,

which does not affect the cross-correlation but achieves desirable temporal properties for radar range resolution. For rectangular arrays, the complex exponentials can be replaced by Kronecker products of complex exponentials, to spatial coverage over an arbitrary rectangular region of space.

Recall from Section 2 that the HMPAR ambiguity function χ_H is a function of 4 variables: τ , f , θ_1 , and θ_2 . τ is the difference in time delays for two targets, and f is the difference in Doppler frequencies for two targets. θ_1 and θ_2 represent the spatial variables for two targets, and because of the spatial beamforming involved in HMPAR operation, which treats certain angles differently than others, there is no spatial stationarity. For a 1-D arrays, θ_1 and θ_2 are scalar angles; for 2-D arrays they each represent both azimuth and elevation which means that the ambiguity function is a function of 6 variables, difficult to visualize. In an effort to understand the key features of the HMPAR operation, we concentrate here on a 1-D scenario.

We imagine a uniform linear array of 256 transmitters at half-wavelength. This could be configured in any number of ways, *e.g.* 16 subarrays of 16 elements each, 32 of 8 each, 64 of 4 each, etc. For our example here we consider an extreme case in which all 256 elements are driven by 256 different signals.

The signals are chosen using the methodology described above. The length is $N = 1024$ and the frequency offset parameter Δf is chosen to cause the beamwidth to be 1/8 of the full range of electrical angles ($-\pi/8$ to $+\pi/8$). The beam center is set to broadside.

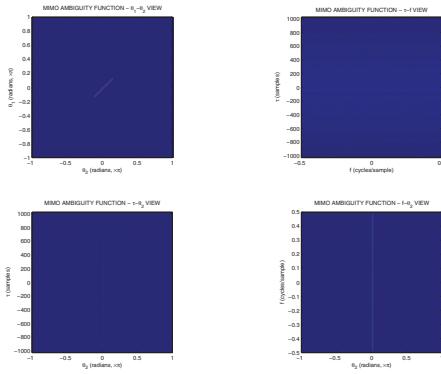


Figure 3.1. MIMO ambiguity function display, 4 views

The ambiguity function was computed using the expressions at the end of Section 2. In order to visualize the ambiguity function, we take 4 different 2-D "slices" through the 4-dimensional function: $\theta_1 - \theta_2$, $\tau - f$, $\tau - \theta_2$, and $f - \theta_2$. All of these are centered at $\chi_H(0, 0, 0, 0)$. In

order the save computation, in preparing this figure only the slices were computed (rather than computing the entire 4-dimensional function and then displaying the slices.)

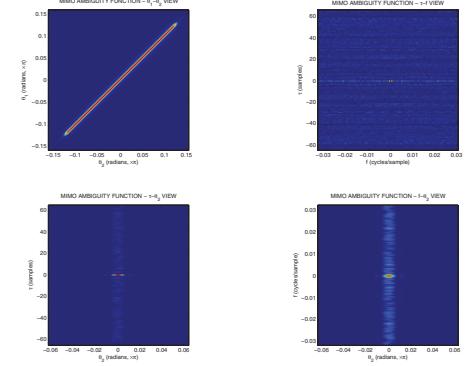


Figure 3.2. MIMO ambiguity function display, 4 views, medium zoom.

These four slices are shown in Figure 3.1. All four images are displayed using a linear scale from 0 to the image maximum with the MATLAB "jet" colormap. In this example the resolution of the ambiguity function is greater than that of the digital images, so little is visible. However, in the upper-left panel, the $\theta_1 - \theta_2$ slice, one can see that the diagonal of the image has support on the interval $[-\pi/8, +\pi/8]$. In fact, the function $\chi_H(0, 0, \theta_1, \theta_2)$ which is the image diagonal, is precisely the transmit beampattern.

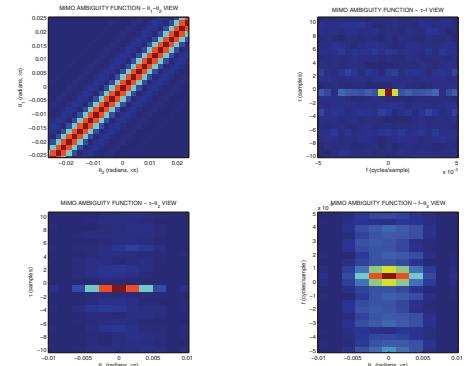


Figure 3.3. MIMO ambiguity function display, 4 views, high zoom.

In Figure 3.2 we show these same four images zoomed toward the center. In Figure 3.3 we zoom to an even higher level of image resolution. From this last image we can make the following observations. First, the resolution in the τ direction is exactly one sample, or equivalently the inverse of the bandwidth of the RF signal. This is a result of our signal design criterion that the discrete-time signal be spectrally flat. Second, the spatial

resolution, as seen in any of the cuts that include θ_2 , is such that the beamwidth on receive is much smaller than the transmit beamwidth. Finally we note that the Doppler sidelobes for this signal are slightly higher than the either the range (time) sidelobes or the spatial sidelobes. This is not surprising in light of the fact that no attempt was made in our signal design to control the Doppler behavior. However, we note in passing that if these signals were to be used in a pulse-Doppler system, then it would be desirable to have low Doppler resolution to avoid issues of Doppler fragility [8]. We leave the exploration of this issue as a topic for future research.

The key message in these graphics, in fact the key message for MIMO radar in general for phase-coherent colocated assets, is this: *the advantage of MIMO radar is its ability to achieve arbitrary spatial beampatterns on transmit, while maintaining full phased-array resolution on receive.* This is a consequence of the fact that MIMO radar systems, even when transmitting broad patterns, *transmit different temporal signals in different directions.* This is what allows the system to differentiate between returns at different angles.

We remark finally that, while the ambiguity function does provide the radar system designer with critical information about radar resolution, it does not address the equally important issue of SNR. It is understood that with broadening of the transmit beam, relative to the maximum directionality of a phased array, there is a loss in SNR that will undoubtedly affect target detection and parameter estimation accuracy. This is not evident in the ambiguity function, as it is based on a nominal transmit power and a nominal target cross-section. The loss in SNR has to be considered along with the transmit beampattern in the overall system design, taking into account any prior knowledge on the target location.

5. CONCLUSION

This paper has the presented one key aspect of our research on various signal strategies which could be employed in the notional HMPAR architecture to achieve various objectives quantified by transmit beampatterns and space-time ambiguity functions. The ambiguity function that is appropriate for the HMPAR radar architecture was derived. Drawing on previous work in transmit signal design, we presented examples of ambiguity functions for these signals using the HMPAR architecture, demonstrating that one can achieve phased-array-like resolution on receive, for arbitrary transmit beampatterns.

Much interesting work remains in the further development of the HMPAR concept. We see this work moving forward in four areas: 1) adaptive processing for clutter

mitigation, 2) constant-modulus signals with desired correlation and ambiguity properties, 3) adaptive transmit beamforming for clutter mitigation, and 4) joint clutter imaging and GMTI detection. It is believed that continuing this line of research will significantly advance the state-of-the-art in next-generation radar systems.

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